UK INTERMEDIATE MATHEMATICAL CHALLENGE

THURSDAY 4TH FEBRUARY 1999

Organised by the **United Kingdom Mathematics Trust from the School of Mathematics, University of Leeds**

SOLUTIONS LEAFLET

This is the first solutions leaflet which has been provided for an IMC. It is sent in the hope that it might provide all concerned with some alternative solutions to the ones they have obtained. It is not intended to be definitive. The organisers would be very pleased to receive alternatives created by candidates.

1. **C** A and B are both less than 199^2 and hence less than 199^9; $9^9 < 199^9$; $1^{999} = 1$.

2. **B** For B to be possible, the diagonal of a rectangle would need to be an axis of symmetry.

3. **D** The maximum number of days in any month is 31 and $31 \times 24 = 744$.

4. **E** Ima's result is ten times bigger than it should be and therefore dividing by 10 will correct her mistake. Multiplying by 0.1 is equivalent to dividing by 10.

5. **D** $30 \div 0.2 = 300 \div 2 = 150$ or $30 \div 0.2 = 30 \div 1/5 = 30 \times 5 = 150$.

6. **A** 250 kg is approximately 25 times 9.6 kg. The consumption in Africa is therefore approximately 25×60, i.e. 1500, per person per year. 4 bananas per day are equivalent to 4×365, i.e. 1460, per year and the best answer is therefore A.

7. **E** The values are respectively ½, 1, ½, ½, ¼.

8. **D** ΔPQR is isosceles and therefore $\angle PQR = \angle PRQ = 72°$. ΔPSR is also isosceles therefore $\angle RPS = \angle RSP = 36°$ (using the external angle theorem or by calculating that $\angle PRS = 108°$) and $x = 180° - 36° - 36° = 108°$.

9. **E** The ratio of height to shadow length is 1:3. Their shadow will therefore be $3 \times (1 + 1.5)\text{m} = 7.5\text{m}$ long.

10. **D** The fine should be approximately $74 \times 52 \times 60\text{p} \approx 75 \times £30 = £2250$.

(*Historical note*: the person who returned the book disappeared without paying a fine at all!)

11. **D** Let the original price be £x. Then $0.8x = 60 \Rightarrow x = 60 \div 0.8 = 75$.

12. **A** Let the hypotenuse be of length h cm. Then $h^2 = 100 + 25 = 125$.

$$\Rightarrow h = \sqrt{125};\ 11^2 = 121 \text{ and } 12^2 = 144 \Rightarrow 11 < h < 12$$

$$1.5^2 = (11 + \tfrac{1}{2})^2 = 11^2 + 2 \times 11 \times \tfrac{1}{2} + \tfrac{1}{4} = 132\tfrac{1}{4} \Rightarrow 11 < h < 11.5$$

$$1.25^2 = (11 + \tfrac{1}{4})^2 = 121 + 2 \times 11 \times \tfrac{1}{4} + \tfrac{1}{16} \Rightarrow 11.25^2 > 126.5 \Rightarrow 11 < h < 11.25$$

The length of the hypotenuse is therefore closer to 11 cm than 11.5 cm.

13. **B** Let the radius of the circle be r. This implies that the radius of the semicircle is $2r$. The area of the semicircle is therefore $\frac{1}{2} \times \pi \times (2r)^2 = 2\pi r^2$ which is twice the area of the circle.

14. **E** The number of diagonals in an n-sided polygon is $\frac{1}{2}n(n - 3)$. This can be used to show that A to D are all correct. A quadrilateral has half as many diagonals as it has sides, not twice as many.

15. C Let AB be of length $3r$. The distance moved by A is then the circumference of a semicircle of radius $3r$ i.e. $3\pi r$. C moves along the circumference of a semicircle of radius $2r$, i.e. $2\pi r$, followed by the circumference of a semicircle of radius r, i.e. πr. The total distance moved by C is therefore also $3\pi r$.

[In fact every point of the pencil moves a total distance of $3\pi r$.]

16. E All powers of 3 are odd and $3^{10} = 3^5 \times 3^5$ which means that 3^{10} is also square.

17. C The area of the largest circle = 25π cm^2.

The shaded area = $(16\pi - 9\pi)\,\text{cm}^2 = 7\pi\,\text{cm}^2$.

The percentage which is shaded = $7/25 \times 100\% = 28\%$.

18. B Let the number of boys in Group I be x. The number of girls in Group I is therefore $40 - x$ and the number of girls in Group II is $33 - (40 - x) = x - 7$. There are therefore 7 girls fewer in Group II than there are boys in Group I.

19. C 1 wuggle = 6 waggles; 2 woggles = 5 waggles; 4 wiggles = 3 woggles= 7.5 waggles. All are therefore greater than 3 waggles.

20. B Two murders: $2x$ hours; 6 car thefts: $6(x/6) = x$ hours; 4 bank robberies: $4(x/2) = 2x$ hours. Total = $(2x + x + 2x)$ hours = $5x$ hours.

21. A The product $= 3/2 \times 4/3 \times 5/4 \times 6/5 \times \ldots \times (n+1)/n$. The numerator of each fraction except the last cancels with the denominator of the next fraction, leaving $(n + 1)/2$. This is equal to an integer only when n is odd.

22. D 206 – the 6 legs of the staff = 200. On average, at each table, there are three table legs, 16 chair legs and 6 customer legs which gives a total of 25 legs per table. $200 \div 25 = 8$, hence there are 8 tables and therefore 32 chairs.

23. E The ten numbers in the star total 75. Each number appears in two "lines" and therefore the five "lines" total 150 which implies the sum of the numbers in each "line" is 30.

This means that $K + C = 24$ and therefore $K = 11$ and $C = 13$ or vice versa.

If $K = 11$ then $U = 12$; $I = 10$ and $M = 11$ which is impossible since $K = 11$.

If $K = 13$ then $U = 10$; $I = 12$; $M = 9$ and $C = 11$ which is correct.

24. A The five knaves all say that a different number ate the tarts which means that only one of them can be telling the truth. K4 is the only honest knave and K1, K2, K3 and K5 ate the tarts.

25. B The fold is made along BE. A folds onto A'.

$A'B = AB = \sqrt{2} \Rightarrow A'C = 1$ (by Pythagoras).

$\Delta A'BC$ is therefore a right-angled isosceles triangle

$$\Rightarrow \angle BA'C = 45° \Rightarrow \angle EA'D = 45°$$

$$\Rightarrow ED = A'D = \sqrt{2} - 1.$$